A Method of Determining the Equivalent Thermal Conductivity of a Conductive Printed Wiring Board (PWB) Layer with a Quasi-Random Circular Hole Pattern

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Abstract

The thermal analysis of Printed Wiring Boards (PWBs) relies on the accurate mathematical representation of the board constituents and the mounted electronic components. The accuracy of the mathematical model of the PWB thermal properties requires knowledge of the equivalent conductivity associated with each of the copper layers. The Universal Conductivity Curve (UCC) for randomly distributed holes in two dimensional sheets, presented in the literature, is based on Percolation Theory and curve fitting electrical conductivity data for various shaped insulation regions. This paper extends this concept to the case of thermal conductivity in thermally conductive sheets and validates a Universal Thermal Conductivity Curve using Finite Element analysis results. An application of the Universal Thermal Conductivity Curve allows the analyst to assess the contribution of each copper layer to PWB thermal conductivity. The Universal Thermal Conductivity Curve represents a method of determining the effective in-plane thermal conductivity ($K_e$) of conducting layers, excluding signal layers, within a PWB. The effective thermal conductivity for each of the conductive layers within a PWB can be determined from the percentage of conductive material associated with the layer. The parallel thermal resistance of each of the conducting layers combines and the PWB equivalent in-plane thermal conductivity is subsequently determined.

Key words:

Equivalent, Thermal Conductivity, Layer, and Printed Wiring Board (PWB).

1. Introduction

The thermal analysis of Printed Wiring Boards (PWBs) relies on the accurate mathematical representation of the printed wiring board constituents and the mounted electronic components. Traditionally, as a means of lowering junction temperatures, metal cores have provided a relatively economical thermal path for the dissipation of component power. The copper layers within the PWB consist of ground and power planes, and signal trace layers. The thermal conductivity of each of these layers provides additional heat flow paths. The effectiveness of the copper layers indigenous to the PWB is secondary to the heat conducted by the metal core to the cooler PWB edges.

In cases where the utilization of a metal core is not an option, the economical thermal management of the circuit card assembly can present an additional design challenge. The accuracy of the mathematical representation of the PWB thermal properties requires knowledge of the conductivity associated with each of the copper layers. In the absence of adequate convection heat transfer, the PWB copper layer conductivity becomes significant in assessing power dissipation to the cold wall through the copper planes within the PWB.

An application of the Universal Conductivity Curve, which describes the electrical conductivity of a two-dimensional continuous sheet containing an array of holes, randomly located, is presented in this work. This application allows the analyst to access the contri-
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2. An Application of Percolation Theory

The UCC, pertaining to sheets with randomly placed circular holes, is based on Percolation Theory. The empirical aspect of the UCC is associated with the selection of coefficients derived from curve fitting electrical conductivity data. This data represented the results associated with various shaped insulation regions within a two-dimensional domain (that is the conductive sheet) and the coefficients are those found in equation (1) (equation (15)). The application of Percolation Theory originated from the question: How does the conductivity of a conductive sheet vary as a function of the random placement of non-conducting regions? An application of Percolation Theory results in determining the affect residual domain connectivity has on the equivalent conductivity of a conductive sheet, resulting from the presence of regular or randomly distributed discontinuities. The solution techniques applied to Percolation models involve algorithms such as the Monte Carlo, Random Walk, or the “Ant in a Labyrinth”. The noteworthy result of Reference is the formulation of a Universal constant.

This constant is given by $x_c = n_c <L_{eff}^2>$, where $n_c$ is the critical value at percolation of the number of holes per unit area, and $L_{eff}$ is an effective length. For circles, $L_{eff}$ is twice the diameter. Furthermore, the electrical conductivity $\sigma(x)$ was shown to be a universal function of $x = n \cdot L_{eff}^2$. The results presented below extend this concept to the case of thermal conductivity in thermally conductive sheets with regular and quasi-random circular hole pattern.

3. Method of Analysis

The applicability of the Universal Conductivity Curve in predicting the effective thermal conductivity ($K_e$), of copper layers in printed wiring boards was determined using the Finite Element method of analysis. The thermal analysis capability of the MSC/NASTRAN, version 70, Finite Element code determined the temperature distribution within the conductive sheets considered. The QUAD4 and CTRIA3 plate elements provided the basis for the mathematical representation of the conductive sheet.

The Finite Element models of the copper conductive sheets were 12.7 mm x 12.7 mm and 0.1524 mm thick, using a conductivity ($k$) of 9.89 W/(in-°C). Figure 1 illustrates the thermal boundary conditions. The lower edge is subjected to a uniform thermal load of 4.747 watts and the upper edge is maintained at a sink temperature of 20°C. The left and right edges are considered adiabatic. Table 1 summarizes the diameter to pitch ratio (D/P), percent residual conductor, the analytical conductivity ratio and the conductivity ratio predicted by the UCC, and the percent error.

The analytical approach was based on Fourier’s Law of Heat Conduction,

$$q = \frac{k \cdot A \cdot \Delta T}{L}$$  \hspace{1cm} (1)

where: $k =$ material thermal conductivity, in W/(in-°C),
$K =$ thermal conductivity of conducting sheet with random hole pattern, W/(in-°C),
$A =$ cross sectional area, mm$^2$,
$L =$ length of conductive path, mm,
$t =$ sheet thickness, mm,
$\Delta T =$ temperature difference ($T_i - T_o$), °C.

Table 1. Hole pattern results.

<table>
<thead>
<tr>
<th>CASE</th>
<th>Diameter pitch ratio (D/P)</th>
<th>% Residual conductor</th>
<th>Analytical K/Ko</th>
<th>UCC CURVE K/Ko</th>
<th>% ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.1</td>
<td>99.21</td>
<td>0.9840</td>
<td>0.9844</td>
<td>0.04</td>
</tr>
<tr>
<td>B</td>
<td>0.25</td>
<td>95.09</td>
<td>0.9101</td>
<td>0.9047</td>
<td>0.60</td>
</tr>
<tr>
<td>C</td>
<td>0.50</td>
<td>80.37</td>
<td>0.6689</td>
<td>0.6529</td>
<td>2.45</td>
</tr>
<tr>
<td>D</td>
<td>0.70</td>
<td>61.52</td>
<td>0.4233</td>
<td>0.4001</td>
<td>5.80</td>
</tr>
<tr>
<td>E</td>
<td>0.90</td>
<td>36.38</td>
<td>0.1716</td>
<td>0.1740</td>
<td>0.29</td>
</tr>
<tr>
<td>F</td>
<td>0.897</td>
<td>36.80</td>
<td>0.1863</td>
<td>0.1740</td>
<td>7.10</td>
</tr>
<tr>
<td>G</td>
<td>0.532</td>
<td>77.78</td>
<td>0.6000</td>
<td>0.6137</td>
<td>2.23</td>
</tr>
</tbody>
</table>
Since the conductive sheets modeled were square, 
\[ A = L \cdot t \], and 
\[ q = K \cdot t \cdot \Delta T \]  \hspace{1cm} (2)

\[ q = K \cdot t \cdot (T_i - T_o) \]  \hspace{1cm} (3)

\[ q = K \cdot t \cdot \Delta T \]

or

From Fourier’s Law of Heat Conduction, the conductivity (K) of a conducting sheet with a regular or random distribution of holes is determined from the power uniformly applied at one edge, the sink temperature at the opposite edge \( T_o \), and the computed temperature \( T_i \), and is given by
\[
K = \frac{q}{(T_i - T_o) \cdot t} \]  \hspace{1cm} (5)

The Universal Conductivity Curve\(^1\) abscissa is the non-dimensional quantity \( x = 4nD^2 \). The hole pitch \( (P) \) is given by \( \frac{D}{\sqrt{n}} \) and the Diameter to Pitch Ratio \( (D/P) \) is of the form,
\[
\frac{D}{P} = \frac{D}{\sqrt{n}} \]  \hspace{1cm} (6)

\[
\frac{1}{P} = \sqrt{n} \]  \hspace{1cm} (7)

where \( n \) = the number of holes per unit area

Solving for \( D \) results in the following relation,
\[
D = (\frac{q}{\pi}) \cdot \sqrt{n} \]  \hspace{1cm} (8)

Substituting into the equation \( x = 4nD^2 \), provides the following,
\[
x = 4n \left[ \left( \frac{D}{P} \right)^2 \cdot \left( \frac{1}{n} \right) \right] = 4 \cdot \left( \frac{D}{P} \right)^2 = \beta \]  \hspace{1cm} (9)

The Universal Conductivity Curve\(^1\), after substitution, takes the form,
\[
\frac{\sigma}{\sigma_o} = \frac{K}{K_o} \left[ 1 - \frac{\beta}{5.90} \left( 1 + \frac{\beta}{5.90} - \frac{\beta^2}{24.97} \right) \right]^{1.3} \]  \hspace{1cm} (10)

\( \sigma \), and \( K \), represent the electrical and thermal conductivity of a continuous conductive sheet, respectively, in the absence of holes or discrete regions of insulation.

The percentage of conductor was determined by dividing the product of the area per element and number of elements by the total area of the conductive sheet, of the form,
\[
\% \text{ Conductor} = \frac{\text{No of Elements} \cdot \text{Area}}{\text{Total Area}} \]  \hspace{1cm} (11-a)

\[
\% \text{Conductor} = 1 - \frac{\pi}{4} \cdot \left( \frac{D}{P} \right)^2 \]  \hspace{1cm} (11-b)

In practice, the Gerber files contain the information required by software packages to determine the percentage of conducting material.

4. The Effect of a Thermal Point Source (\( \cdot \)) in Determining Effective Thermal Conductivity

The electrically conductive paths within a network include connections from sources of power, to ground and power planes. The transport of heat by conduction and associated with component power dissipation, flows along electrically conductive paths and partially diffuses into the conductive planes within the PWB. A routing algorithm determines the entry point for an electrically conductive path, and consequently thermal energy, into a conductive plane. The locations of the entry point into the conductive plane tend to be pseudo-random as a result of the optimization function of the routing algorithm.

The heat dissipated by electronic components conducts through the component leads to the PWB. This effect was evaluated analytically by the application of a point source of heat into a conductive plane. Figure 2 illustrates this case. The four holes in the upper left are 0.3860 mm in diameter. The powers dissipated at these locations are 0.5626 W, 0.5450 W, 0.5626 W, and 0.5450 W, respectively, moving from left to right in a row wise direction from the top. The two holes at the lower edge are 0.3352 mm in diameter, and in a similar manner the power dissipated at each of these locations is 0.4571 W. The remaining holes to the right are 0.1828 mm in diameter, and in a similar manner the power dissipated at each of these locations are 0.3860 mm in diameter, and in a similar manner the power dissipated at each of these locations is 0.2637 W, 0.2813 W, 0.2461 W, 0.2461, 0.2813 W, 0.2461 W, and 0.2637 W, respectively. The total power applied is 4.9577 W. The reference conductivity, \( K_o = 23.42 \text{ W/(in }^2\text{-°C}) \), was determined by applying these point heat sources to a continuous sheet at the locations identified in Figure 2. Figure 3 depicts the predicted temperature distribution in a continuous sheet in response to point heat sources. The apparent hole positions, shown in Figure 3, are displayed to orient the reader for comparison to Figure 4. Figure 4 displays the predicted temperature distribution associated with a sheet containing holes and having discrete heat sources applied on the circumference of the selected holes shown in Figure 2. The thermal conductivity for the configuration of Figure 4 was calculated \( K = 13.98 \text{ W/(in }^2\text{-°C}) \), by the previous methods. The resulting \( K/K_o \) of 0.5969 is plotted in Figure 5. When compared to the Universal Conductivity Curve value of 0.6137, the calculated value is 2.74%
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greater. This alternative approach suggests that the conductivity ratio is an invariant; whether the power dissipation is uniform or discrete.

Figure 2. Discrete (~0.2 to 0.5 W) application of point source heat (Total power applied = 4.95 W).

Figure 3. Temperature distribution associated with discrete applications of power to a continuous sheet (see Figure 2).

Figure 4. Temperature distribution associated with discrete applications of power to a sheet with hole (see Figure 2).

Figure 5. Universal Conductivity Curve: Finite Element predictions.

5. Experimental Results

The results are summarized in Table 1 and Figure 5. Figures 5, inserts (A) through (G), illustrate the hole geometry for each case. Case A through G fall within 7 percent of the Universal Conductivity Curve. Figure 5 indicates the relationship between the Finite Element results and the Universal Conductivity Curve predictions. Each of the hole patterns reference a data point. The hole pattern depicted in Figure 5 (G) was modeled to represent a section of an actual PWB. In addition, planar thermal conductivity test results of a 25 mm x 76 mm coupon, cut from a 16 layer SMT Printed Wiring Board, were predicted with the Universal Thermal Conductivity Curve and equation (3.6), Reference 4. The percent copper coverage at each layer for the test coupon was determined from the PWB artwork at the location of the cut that defined the coupon. The conductivity test measurement of 17.0 W/m°C compared to within 3% of the Universal Thermal Conductivity Curve prediction. The result of Reference 4 was 57% higher than the test result.

6. Conclusions

Figure 6 represents the Universal UCC in two inter-related forms. The dashed curve represents a functional relationship between the diameter to pitch ratio associated with a PWB layer and the ratio (K/Ko). The solid curve represents a functional relationship between the percentage of conducting material in a PWB layer and the ratio (K/Ko). This ratio is a conductivity reduction factor (Kr) applied to the conducting material. For instance, a sheet with 68.9 percent copper would result in a reduction factor of 0.50. Assuming this represented one layer in a PWB, the thermal model of this layer
would use an effective conductivity based on the product of $K_r$ and the conductivity of the conducting material ($k$). For copper, this would result in the following relation,

$$ Ke = 0.50 \times 9.89 \frac{W}{in} - C = 4.945 \frac{W}{in} - C $$

(12)

The conductivity reduction factor can be determined from either the percentage copper within a conductive PWB layer, or a known diameter to pitch ratio. The relationship between $(D/P)$ and percent Conductor can also be found from Figure 6.

![Figure 6. Universal Thermal Conductivity Curve: Diameter to Pitch Ratio (D/P) and Percent Conductor versus the ratio of $K/K_o$](image)

The Universal Conductivity Curve represents a method of determining the effective in-plane thermal conductivity of conducting layers within a PWB, excluding signal layers. The effective thermal conductivity for each of the conductive layers within a PWB can be determined from the percentage of conductive material associated with the layer. An overall in-plane conductivity for the PWB is arrived at mathematically. The parallel thermal resistance of each of the conducting layers combines and the equivalent in-plane thermal conductivity is subsequently determined.

### References


### About the author

Mark Palie received the B.S. Degree in Mathematics from Lowell Tech, and the M. S. Degree in Mechanical Engineering from the University of Lowell. He is currently a Principal Engineer with the mechanical analysis group at Sanders – a Lockheed Martin Company. He has been employed with Lockheed Martin for 10 years and is currently involved with structural and heat transfer analysis. His recent work has involved the nonlinear analysis and fatigue life predictions of solder joints for a variety of surface mounted components.

### 7. Recommendations

- Establish additional validation of these results using experimental data.
- Extend this approach to determine the conductivity normal ($K_z$) to the conductive plane accounting for the thermal coupling between vias (thermal and electrical) and each of the conductive sheets within the PWB.
- Develop a Finite Element model of a typical signal layer and extend these results to include a method of establishing an equivalent conductivity of a signal layer.