Analysis of Multidielectric Multiconductor Hybrid Circuits Using a Closed-Form MoL Approximation

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Abstract

In this paper, the quasi-static parameters of planar multilayer structures are determined using closed-form approximations, where the thickness of the metal strips is considered. This improves the zero thickness analysis used previously by many authors. The developed algorithm is based on the method of lines. The technique is well suited for planar structures. Compared to other methods, it is simpler, accurate, and has no convergence problem. Its semianalytical nature allows saving significant memory and computation time. A derivation of the recursive relations is presented and applied to a set of typical hybrid circuit elements having one, two, and four metal strips. The results obtained are compared with other previously published data.

Key words:
Hybrid Circuits, Quasi-Static, Method of Lines, and MoL.

1. Introduction

Hybrid circuits are considered to be a cost effective technology due to their many advantages such as small size, package cost reduction, power consumption, and reliability. Each particular layer in the multilayered architecture of the hybrid device is added to improve its electric performance. This fact is clearly evident from many publications. The quasi-static analysis used to extract the electric parameters for such circuits provides the engineer with reliable data for the design of hybrid microwave circuits, in particular in the low gigahertz frequency range. In this context, many quasi-static investigations on multilayer structures have been performed using different mathematical techniques. The most popular ones are the Finite Difference and the Finite Element methods, the integral equation approach such as the method of moments and the boundary element method. The method of lines (MoL), which is a differential-difference technique, is a versatile seminumerical technique developed by R. Pregla and co-workers for the analysis of planar microwave structures. It allows an analytical solution in one dimension. Its semi-analytical nature makes its computational effort much less than the other cited methods for the same analyzed structures. In addition, the MoL has no problem of relative convergence.

Among the above techniques used to determine the parameters of multilayered structures, only few have taken into account the effect of strips metallization thickness. For instance, the boundary element method has been applied to shielded microstrip line and showed good agreement with both the variational and...
the current density approximation techniques\(^{10,11}\). The method of lines has been used by many authors for the analysis of microwave structures and has been applied to shielded planar multi-conductor line systems with vanishing metal strip thickness\(^{9,12,13}\). This method has been also extended to the case of finite metal thickness as well\(^{7}\). However, the relatively cumbersome analysis presented in Reference\(^7\) involves extra effort in both mathematical and computational analysis with respect to the approach proposed in this work. The work presented in Reference\(^7\) is based on a fullwave approach developed by Warm\(^{15}\) for the analysis of multilayered two-dimensional structure as shown in Figure 1. It consists of two regions A and B, made of various dielectric layers enclosed in a metallic shielding box. At the interface between regions A and B, corresponding to \(y=0\), \(N\) conductor strips of width \(W_i\) and thickness \(t\) are placed. The thickness of each dielectric layer is taken as \(H_a(z)\) for layers of region A and as \(H_b(z)\) for layers of region B. The dielectric constants are \(\varepsilon_{A(i)}\) and \(\varepsilon_{B(i)}\) for layers A, and \(\varepsilon_{A(B)}\) and \(\varepsilon_{B(B)}\) for layers B, in the \(x\) and \(y\) directions, respectively.

\[
\varepsilon_{\alpha} \frac{\partial^2 \psi(x,y)}{\partial y^2} + \varepsilon_{\alpha} \frac{\partial^2 \psi(x,y)}{\partial x^2} = 0 \quad \text{(1)}
\]

and

\[
\psi(x,y) = 0 \quad \text{(On outer boundary)} \quad \text{(2)}
\]

Other continuity conditions have to be considered at each interface between adjacent layers in regions A and B.

Using the method of lines approach with a non-uniform discretization scheme, Equation (1) can be transformed into a second order system of ordinary differential equations (refer for example to references\(^{12,16}\)). The following system of decoupled ordinary differential equations can thus be obtained,

\[
\frac{d^2 (T\psi)}{dy^2} + \frac{\varepsilon_{\alpha}}{\varepsilon_{\gamma}} \frac{T}{\varepsilon_{\gamma}} \frac{d}{dy} (T\psi) = 0 \quad \text{(3a)}
\]

Or simply,

\[
\frac{d^2 \psi}{dy^2} + \frac{\varepsilon_{\alpha}}{\varepsilon_{\gamma}} \frac{\lambda}{h^2} \psi = 0 \quad \text{(3b)}
\]

where \(D_{\lambda j}\) is a second order difference operator, and \(T\) is an eigenvector matrix, \(\psi\) is a transformed potential vector, and \(h\) is the minimum discretization distance.

The general solution of Equation (3b) in the dielectric layer \(i\) can be written as follows,

\[
V_j^i(y) = C_2 \sinh(\sigma_j^i y) + C_1 \cosh(\sigma_j^i y) \quad \text{(4)}
\]

where, \(\sigma_j^i = (-\varepsilon_{\alpha}/\varepsilon_{\gamma})^{0.5}/h\), \(C_1\) and \(C_2\) are arbitrary constants, and \(\lambda\) is the \(j\)th eigenvalue of the matrix \(D_{\lambda j}\).

For a simplified structure with three dielectric layers as shown in Figure 2, the transformed potential \(V_j\) on the \(j\)th line may be expressed as follows,

\[
V_j^B(1) = A \frac{\sinh[\sigma_j^{B(1)}(y + H_a(1) + H_b(2))]}{\cosh[\sigma_j^{B(1)} H_a(1)]} \quad \text{(5)}
\]

for the layer \(B(1)\),

\[
V_j^{B(1)}(y) = B \frac{\sinh[\sigma_j^{B(1)}(y + H_a(1) + H_b(2))]}{\cosh[\sigma_j^{B(1)} H_a(1)]} + C \frac{\cosh[\sigma_j^{B(1)}(y + H_b(1))] - \cosh[\sigma_j^{B(1)} H_a(1)]}{\cosh[\sigma_j^{B(1)} H_b(1)]} \quad \text{(6)}
\]

for the layer \(B(2)\),

\[
V_j^{B(1)} = D \frac{\sinh[\sigma_j^{B(1)}(y + H_b(1))]}{\cosh[\sigma_j^{B(1)} H_b(1)]} \quad \text{(7)}
\]

for the layer \(B(1)\). Note that \(A, B, C, D\) and \(D\) are constants.

![Figure 1. Cross sectional view of a shielded multilayer hybrid structure with \(N\) metallic strips.](image1)

![Figure 2. A reduced three-dielectric layer structure with \(N\) strip conductors.](image2)
Applying the continuity conditions for the potentials and fields at the different interfaces, one can write at interface between layer A(1) and layer A(2),

$$V_j^{A(1)}(y)_{y=-H_2} = V_j^{A(2)}(y)_{y=-H_2} \quad (8a)$$

and

$$\mathcal{E}_{ny(1)} \frac{dV_j^{A(1)}(y)}{dy}_{y=-H_2} = \mathcal{E}_{ny(2)} \frac{dV_j^{A(2)}(y)}{dy}_{y=-H_2} \quad (8b)$$

Replacing $V_j^{A(1)}$ and $V_j^{A(2)}$ by their expressions given by Equations (5), and (6), respectively, Equations (8a) and (8b) become,

$$A \tan[h \sigma^{A(1)}_j H_a(1)] = \frac{C}{\cosh[\sigma^{A(2)}_j H_a(2)]]} \quad (9a)$$

and

$$\mathcal{E}_{ny(1)} A = \mathcal{E}_{ny(2)} \frac{B}{\cosh [\sigma^{A(2)}_j H_a(2)]]} \quad (9b)$$

Note that at the interface A(2) / B(1), one may distinguish two different regions; the regions which contain conductor strips and the regions which do not contain any conductor strip. For the regions with no conductors, the continuity conditions may be expressed as follows,

$$V_j^{A(2)}(y)_{y=0} = V_j^{B(1)}(y)_{y=0} \quad (10a)$$

and

$$\mathcal{E}_{ny(2)} \frac{dV_j^{A(2)}(y)}{dy}_{y=0} - \mathcal{E}_{ny(1)} \frac{dV_j^{B(1)}(y)}{dy}_{y=0} = 0 \quad (10b)$$

Making use of Equations (6) and (7), Equations (10a) and (10b) become,

$$B \tan[h \sigma^{A(2)}_j H_a(2)] + C = -D \tan[h \sigma^{B(1)}_j H_b(1)] \quad (11a)$$

$$\mathcal{E}_{ny(2)} \sigma^{A(2)}_j \{B + C[\tan[h \sigma^{A(2)}_j H_a(2)]]\} - \mathcal{E}_{ny(1)} \sigma^{B(1)}_j D = 0 \quad (11b)$$

The thickness $t$ of the metal strips should be included in the different potential and field expressions for the interfaces having conductor strips. Assuming the voltage on the metal strips to be constant, such that,

$$V_j^{B(1)}(0) = V_j^{B(1)}(t) \quad (12)$$

This allows writing the continuity conditions on potentials and fields as follows,

$$V_j^{A(2)}(y)_{y=t} = V_j^{B(1)}(y)_{y=t} = V_j(0) \quad (13a)$$

$$\mathcal{E}_{ny(2)} \frac{dV_j^{A(2)}(y)}{dy}_{y=t} - \mathcal{E}_{ny(1)} \frac{dV_j^{B(1)}(y)}{dy}_{y=t} = S_j \quad (13b)$$

Which in turn can be rewritten under the following forms,

$$B \tan[h \sigma^{A(2)}_j H_a(2)] + C = D \frac{\sinh[\sigma^{B(1)}_j (t - H_b)]}{\cosh[\sigma^{B(1)}_j H_b(1)]} = V_j(0) \quad (14a)$$

$$\mathcal{E}_{ny(2)} \sigma^{A(2)}_j \{B + C \tan[h \sigma^{A(2)}_j H_a(2)]\} - \mathcal{E}_{ny(1)} \sigma^{B(1)}_j D = S_j \quad (14b)$$

Eliminating all arbitrary constants from the above equations, one can end up with the following equations,

$$V_j(0) \sigma^{A(2)}_j \{\mathcal{E}_{ny(2)} \xi + \frac{\mathcal{E}_{ny(1)}}{\cosh[\sigma^{A(2)}_j H_a(1)]] + \sinh(-\sigma^{A(2)}_j t)}{\cosh[\sigma^{A(2)}_j H_a(1)]] + \sinh(-\sigma^{A(2)}_j t)} \} = S_j \quad (15a)$$

$$V_j(0) \sigma^{A(2)}_j \{\mathcal{E}_{ny(2)} \xi + \frac{\mathcal{E}_{ny(1)}}{\cosh[\sigma^{A(2)}_j H_a(1)]] + \sinh(-\sigma^{A(2)}_j t)}{\cosh[\sigma^{A(2)}_j H_a(1)]] + \sinh(-\sigma^{A(2)}_j t)} \} = S_j \quad (15b)$$

respectively, in the regions without and with metallic strips, where,

$$\xi = \frac{\mathcal{E}_{ny(1)}}{\cosh[\sigma^{A(2)}_j H_a(1)]] + \sinh(-\sigma^{A(2)}_j t)}{\cosh[\sigma^{A(2)}_j H_a(1)]] + \sinh(-\sigma^{A(2)}_j t)} \quad (16)$$

and

$$C_j V_j = hS_j \quad (17)$$

where $C_j$ are proportionality factors. For the structure of Figure 1, the expression of the proportionality constants $C_j$ can be expressed as follows,

$$C_j = h(\sigma^{z=0} C_a (Az) + \sigma^{z=0} C_b (Bz)) \quad (18)$$

or,

$$C_j = h(\sigma^{z=0} C_a (Az)) + \frac{\mathcal{E}_{ny(1)} \sigma^{B(1)}_j C_b (Bz)}{\mathcal{E}_{ny(1)} \cos[\sigma^{z=0} H_b(1)] + C_a (Bz) \sinh(-\sigma^{z=0} H_b(1))} \quad (19)$$

respectively, in the regions without and with metallic strips where,

$$C_a(z) = \mathcal{E}_{ny(1)} \frac{C_a(z-1) + \mathcal{E}_{ny(1)} \tan[h \sigma^{A(2)}_j H_a(z)]}{\mathcal{E}_{ny(1)} + C_a(z-1) \tan[h \sigma^{A(2)}_j H_a(z)]} , z=2,3,... \text{Az.} \quad (20)$$

with

$$C_a(1) = \frac{\mathcal{E}_{ny(1)}}{\cosh[\sigma^{A(1)}_j H_a(1)]]} \quad (21)$$

and

$$C_a(z) = \frac{C_a(z-1) + \mathcal{E}_{ny(1)} \tan[h \sigma^{A(2)}_j H_a(z)]}{\mathcal{E}_{ny(1)} + C_a(z-1) \tan[h \sigma^{A(2)}_j H_a(z)]} , z=2,3,... \text{Bz.} \quad (22)$$

On conducting strips, at $y=0$ and $y=t$, the charge density that belongs to the $j$th line is given by $S_j$, which is related to the charge $q_j$ by the following relationship,

$$S_j = q_j/e_j \quad (24)$$
In most hybrid microwave integrated circuits, the metal strip thickness represents at most 30% of the metal strip width. Therefore, the charge on the lateral side of the metallic strips has been neglected in this analysis. After transforming, normalizing and reducing to the original domain, Equation (24) leads to the following,

\[ q_{red} = \gamma_{red}^{-1} \cdot I \]  

From which the capacitance matrix of the analyzed structure can be derived by summing up all the sub-matrices.

3. Results and Discussions

Based on the developed approach, a computer program has been written and three shielded structures were simulated and some results are compared to previously published data. These structures are the microstrip line, the coupled line, and a five dielectric layer structure with four metallic strips. The obtained results are reported in Tables 1, 2, 3, and 4.

Table 1. Values of the characteristic impedance for a shielded microstrip line obtained with \( M_{min} = 5 \).

<table>
<thead>
<tr>
<th>t (mm)</th>
<th>W (mm)</th>
<th>Ratio ( t/W )</th>
<th>Zo (Ω) Ref. [6]</th>
<th>Zo (Ω) (This work)</th>
<th>Relative Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.2</td>
<td>0.25</td>
<td>121.14</td>
<td>120.92</td>
<td>0.18 %</td>
</tr>
<tr>
<td>0.10</td>
<td>0.2</td>
<td>0.5</td>
<td>108.96</td>
<td>109.01</td>
<td>0.05 %</td>
</tr>
<tr>
<td>0.20</td>
<td>0.2</td>
<td>1.0</td>
<td>92.44</td>
<td>91.64</td>
<td>1.98 %</td>
</tr>
<tr>
<td>0.20</td>
<td>0.4</td>
<td>0.5</td>
<td>80.92</td>
<td>76.53</td>
<td>5.42 %</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>0.33</td>
<td>50.54</td>
<td>41.19</td>
<td>17.15 %</td>
</tr>
</tbody>
</table>

Table 2. Capacitance matrix elements of a coupled line structure using different methods.

<table>
<thead>
<tr>
<th>t (mm)</th>
<th>C_{11} / \varepsilon_0</th>
<th>C_{12} / \varepsilon_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>14.61</td>
<td>-7.06</td>
</tr>
<tr>
<td>0.04</td>
<td>15.20</td>
<td>-7.73</td>
</tr>
<tr>
<td>0.06</td>
<td>15.69</td>
<td>-8.28</td>
</tr>
<tr>
<td>0.08</td>
<td>16.34</td>
<td>-8.99</td>
</tr>
<tr>
<td>0.10</td>
<td>17.23</td>
<td>-10.0</td>
</tr>
<tr>
<td>0.12</td>
<td>18.48</td>
<td>-11.4</td>
</tr>
</tbody>
</table>

Table 3. Effect of strip thickness on even and odd mode characteristic impedances of a coupled strip anisotropic structure.

<table>
<thead>
<tr>
<th>t (mm)</th>
<th>Ze (Ω)</th>
<th>Zo (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>110.16</td>
<td>54.69</td>
</tr>
<tr>
<td>0.05</td>
<td>107.75</td>
<td>50.55</td>
</tr>
<tr>
<td>0.15</td>
<td>102.87</td>
<td>47.08</td>
</tr>
<tr>
<td>0.20</td>
<td>100.82</td>
<td>44.22</td>
</tr>
</tbody>
</table>

Table 4. Values of capacitance matrix elements of a five-dielectric structure with four conductors.

<table>
<thead>
<tr>
<th>t (mm)</th>
<th>C_{11}</th>
<th>C_{12}</th>
<th>C_{13}</th>
<th>C_{14}</th>
<th>C_{22}</th>
<th>C_{23}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>14.61</td>
<td>-7.06</td>
<td>-0.84</td>
<td>-0.21</td>
<td>18.13</td>
<td>-6.67</td>
</tr>
<tr>
<td>0.02</td>
<td>14.84</td>
<td>-7.33</td>
<td>-0.74</td>
<td>-0.28</td>
<td>18.65</td>
<td>-6.99</td>
</tr>
<tr>
<td>0.04</td>
<td>15.20</td>
<td>-7.73</td>
<td>-0.61</td>
<td>-0.36</td>
<td>19.44</td>
<td>-7.44</td>
</tr>
<tr>
<td>0.06</td>
<td>15.69</td>
<td>-8.28</td>
<td>-0.48</td>
<td>-0.47</td>
<td>20.54</td>
<td>-8.01</td>
</tr>
<tr>
<td>0.08</td>
<td>16.34</td>
<td>-8.99</td>
<td>-0.32</td>
<td>-0.61</td>
<td>21.92</td>
<td>-8.71</td>
</tr>
<tr>
<td>0.10</td>
<td>17.23</td>
<td>-10.0</td>
<td>-0.08</td>
<td>-0.82</td>
<td>23.79</td>
<td>-9.65</td>
</tr>
<tr>
<td>0.12</td>
<td>18.48</td>
<td>-11.4</td>
<td>0.34</td>
<td>-1.20</td>
<td>26.03</td>
<td>-11.0</td>
</tr>
</tbody>
</table>

For the shielded microstrip line, the structure parameters are \( H_a(1)+H_b(1)=1 \), \( r_a(1)=r_b(1)=1.0 \), \( S_1+S_2+W=1 \). The results obtained for this structure are summarized in Table 1 together with the ones obtained using the boundary element method. It can be seen from this Table that the characteristic impedance of the structure obtained by the two methods compare well (relative difference less than 2%) for small values of strip width \( W \). When the strip width \( W \) increases for fixed enclosure dimensions, that is, the ratio \( W/(s_1+s_2+W) \) increases, the relative difference in the two results increases. The error is in fact due to neglecting the strip lateral side charges in this case. However, if this ratio decreases, the two results compare very well. In practical situation for most hybrid circuits, the enclosure dimensions are either infinite (open structures) or much larger than the strip width. In this case, the approach presented in this work is more efficient since it is simpler to implement, requires less memory and computing time, and provides results comparable to other techniques.

The parameters for a coupled line structure for \( N=2 \), \( W_1=W_2=3 \), \( H_a(1)=1 \), \( H_b(1)=4 \), \( S_1=S_3=5 \), \( S_2=2 \), \( e_{\text{air}}=\varepsilon_{\text{air}}=1.0 \) and \( t=1 \) are shown in Table 2. They represent the values of the capacitance matrix elements \( C_{11}/\varepsilon_0 \) and \( C_{12}/\varepsilon_0 \), which are compared with other data given in References for similar structure parameters. The effect of metallization thickness \( t \) on odd and even mode parameters of anisotropic structures has also been studied. The parameters of the structure considered are \( N=2 \), \( W_1=W_2=0.5 \), \( H_a(1)=1 \), \( H_a(2)=0.5 \), \( H_b(1)=10 \), \( S_1=S_3=1 \), \( S_2=0.12 \) and the dielectric con-
Analysis of Multidielectric Multiconductor Hybrid Circuits Using a Closed-Form MoL Approximation

The capacitance matrix of planar multilayered structures has been determined using closed-form approximation based on the method of lines. The thickness of metallic conductor strips embedded within the structures is considered for better accuracy with respect to the zero thickness approach. The developed algorithm, based on the MoL, which has a semianalytical nature, allows saving significant memory and computation time. This facilitates its integration into current CAD tools for hybrid circuits. However, the presented analysis is limited to multilayered structures where the ratio of strip thickness to strip width is small (less than 0.2). To improve the accuracy for thick conductors, the boundary conditions at the sidewalls of the metal strips have to be considered.

4. Conclusion

The capacitance matrix of planar multilayered structures has been determined using closed-form approximation based on the method of lines. The thickness of metallic conductor strips embedded within the structures is considered for better accuracy with respect to the zero thickness approach. The developed algorithm, based on the MoL, which has a semianalytical nature, allows saving significant memory and computation time. This facilitates its integration into current CAD tools for hybrid circuits. However, the presented analysis is limited to multilayered structures where the ratio of strip thickness to strip width is small (less than 0.2). To improve the accuracy for thick conductors, the boundary conditions at the sidewalls of the metal strips have to be considered.

References


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